Waiting for Clear Signals of New Physics in B and K Decays

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Abstract

We classify the extensions of the Standard Model (SM) according to the structure of local operators in the weak effective Hamiltonian and the presence or absence of new flavour and CP-violating interactions beyond those represented by the CKM matrix. In particular we review characteristic properties of models with minimal flavour violation (MFV), models with significant contributions from Higgs penguins and models with enhanced Z^0 penguins carrying a large new CP-violating phase. Within the latter models, the anomalous behaviour of certain $B \to \pi K$ observables implies large departures from the SM predictions for rare and CP-violating K and B decays. Most spectacular is the enhancement of $Br(K_{\mathrm{L}} \to \pi^0 \nu \bar{\nu})$ by one order of magnitude and a strong violation of the MFV relation $(\sin 2\beta)_{\pi\nu\bar{\nu}} = (\sin 2\beta)_{\psi K_S}$. On the other hand our prediction for $(\sin 2\beta)_{\phi K_S} \approx 0.9$ differs from the Belle result by the sign but is consistent with the BaBar value. We give a personal shopping list for the coming years.

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1 Introduction

The quark flavour dynamics of the Standard Model (SM) is consistent with the existing data on K and B meson decays within experimental and theoretical uncertainties. In spite of this, most of us expect that when the precision of experiments and also the theoretical tools improve, some clear signals of new physics (NP) at very short distance scales will be seen. As the search for new phenomena through K and B decays is necessarily an indirect one, it will not be easy to find out what precisely this NP is. This will be even the case in the presence of clear deviations from the SM expectations.

Yet, remembering that decays like $K_L \to \mu^+\mu^-$, $K_L \to \pi\pi$ and the $K^0 - \bar{K}^0$ mixing played an important role in the construction of the SM [1] and led to the GIM mechanism [2] and the CKM matrix [3], we are confident that these indirect signals of NP will give us definitive hints where to go and where not to go. This in turn will allow us to select few competing theories which will be tested directly through high energy collider experiments. Moreover, these signals will hopefully give us additional hints for the fundamental flavour dynamics at ultra short distance scales that cannot be tested directly in a foreseeable future. It is obvious that for this project to become successful, it is essential to

- make the SM predictions for K and B decay observables as accurate as possible, in order to be sure that the observed deviations from the SM expectations originate in NP contributions and are not due to our insufficient understanding of hadron dynamics and/or truncation of the perturbative series.
- consider simultaneously as many processes as possible. Only in this manner the paramaters of a given theory can be fully determined and having them at hand predictions for other observables can be made. In this enterprise correlations between various observables play an important rôle, as they may exclude or pinpoint a given extension of the SM even without a detailed knowledge of the parameters specific to this theory.

- consider observables which while being sensitive to the short distance structure of the theory are only marginally sensitive to the long distance dynamics that is not yet fully under control at present.

These notes review selected aspects of this program. Due to severe space limitations I will follow the strategy of many of my colleagues and will concentrate to a large extent on my own work and the work done in my group in Munich. In Section 2, after giving a master formula for weak decay amplitudes, I will group various extentions of the SM in five classes. In subsequent three sections, I will discuss the first three classes that, being more predictive than the remaining two, should be easiest to test. A shopping list in Section 6, with the hope to be able to distinguish between various NP scenarios, ends this presentation. Recent more detailed reviews can be found in [4].

2 Theoretical Framework

2.1 Master Formula for Weak Decays

The present framework for weak decays is based on the operator product expansion that allows to separate short (μ_{SD}) and long (μ_{LD}) distance contributions to weak amplitudes and on the renormalization group (RG) methods that allow to sum large logarithms $\log \mu_{SD}/\mu_{LD}$ to all orders in perturbation theory. The full exposition of these methods can be found in [5,6].

An amplitude for a decay of a meson $M = K, B, ..$ into a final state $F = \pi\nu\bar{\nu}$, $\pi\pi$, πK is then simply given by

$$A(M \to F) = \frac{G_F}{\sqrt{2}} \sum_i V^i_{CKM} C_i(\mu) \langle F|Q_i(\mu)|M \rangle. \tag{1}$$

Here G_F is the Fermi constant and Q_i are the relevant local operators which govern the decays in question. They are built out of quark and lepton fields. The Cabibbo-Kobayashi-Maskawa factors V^i_{CKM} [3], the matrix elements $\langle F|Q_i(\mu)|M \rangle$ and the Wilson coefficients $C_i(\mu)$, evaluated at the renormalization scale μ, describe the strength with which a given operator enters the amplitude.

Formula (1) can be cast into a master formula for weak decay amplitudes that goes beyond the SM [7]. It reads (we suppress G_{F}):

$$A(\mathrm{Decay}) = \sum_i B_i \eta^i_{\mathrm{QCD}} V^i_{\mathrm{CKM}} [F^i_{\mathrm{SM}} + F^i_{\mathrm{New}}]$$
$$+ \sum_k B^{\mathrm{New}}_k [\eta^k_{\mathrm{QCD}}]^{\mathrm{New}} V^k_{\mathrm{New}} [G^k_{\mathrm{New}}]. \tag{2}$$

The non-perturbative parameters B_i represent the matrix elements of local operators present in the SM. For instance in the case of $K^0 - \bar{K}^0$ mixing, the matrix element of the operator $\bar{s}\gamma_\mu(1 - \gamma_5)d \otimes \bar{s}\gamma^\mu(1 - \gamma_5)d$ is represented by

the parameter $\hat{B}_K$. There are other non-perturbative parameters in the SM that represent matrix elements of operators Q_i with different colour and Dirac structures. The objects η^i_{QCD} are the QCD factors resulting from RG-analysis of the corresponding operators and F^i_{SM} stand for the so-called Inami-Lim functions [8] that result from the calculations of various box and penguin diagrams in the SM. They depend on the top-quark mass.

New physics can contribute to our master formula in two ways. It can modify the importance of a given operator, present already in the SM, through the new short distance functions F^i_{New} that depend on the new parameters in the extensions of the SM like the masses of charginos, squarks, charged Higgs particles and $\tan\beta = v_2/v_1$ in the MSSM. These new particles enter the new box and penguin diagrams. In more complicated extensions of the SM new operators (Dirac structures), that are either absent or very strongly suppressed in the SM, can become important. Their contributions are described by the second sum in (2) with $B^{\mathrm{New}}_k, [\eta^k_{\mathrm{QCD}}]^{\mathrm{New}}, V^k_{\mathrm{New}}, G^k_{\mathrm{New}}$ being analogs of the corresponding objects in the first sum of the master formula. The V^k_{New} show explicitly that the second sum describes generally new sources of flavour and CP violation beyond the CKM matrix. This sum may, however, also include contributions governed by the CKM matrix that are strongly suppressed in the SM but become important in some extensions of the SM. In this case $V^k_{\mathrm{New}} = V^k_{\mathrm{CKM}}$. Clearly the new functions F^i_{New} and G^k_{New} as well as the factors V^k_{New} may depend on new CP violating phases complicating considerably phenomenological analyses.

2.2   Classification of New Physics

Classification of new physics contributions can be done in various ways. Having the formula (2) at hand let us classify these contributions from the point of view of the operator structure of the effective weak Hamiltonian, the complex phases present in the Wilson coefficients of the relevant operators and the distinction whether the flavour changing transitions are governed by the CKM matrix or by new sources of flavour violation [9]. For the first four classes below we assume that there are only three generations of quarks and leptons. The last class allows for more generations.

Class A

This is the simplest class to which also the SM belongs. In this class there are no new complex phases and flavour changing transitions are governed by the CKM matrix. Moreover, the only relevant operators are those that are relevant in the SM. Consequently NP enters only through the Wilson coefficients of the SM operators that can receive new contributions through diagrams involving new internal particles.

The models with these properties will be called Minimal Flavour Violation (MFV) models, as defined in [10]. Other definitions can be found in [11,12]. In this case our master formula simplifies to

$$\mathrm{A(Decay)} = \sum_i B_i \eta^i_{\mathrm{QCD}} V^i_{\mathrm{CKM}} F_i(v) \qquad F_i = F^i_{\mathrm{SM}} + F^i_{\mathrm{New}} \ \ (\text{real}), \qquad (3)$$

where $F_i(v)$ are the *master functions* of MFV models [13]

$$S(v), \ X(v), \ Y(v), \ Z(v), \ E(v), \ D'(v), \ E'(v) \qquad (4)$$

with v denoting collectively the parameters of a given MFV model. A very detailed account of MFV can be found in [13]. In Section 3 some of its main features will be recalled. Examples of models in this class are the Two Higgs Doublet Model II and the constrained MSSM if $\tan\beta$ is not too large. Also models with one extra universal dimension are of MFV type.

Class B

This class of models differs from class A through the contributions of new operators not present in the SM. It is assumed, however, that no new complex phases beyond the CKM phase are present. We have then

$$\mathrm{A(Decay)} = \sum_i B_i \eta^i_{\mathrm{QCD}} V^i_{\mathrm{CKM}} [F^i_{\mathrm{SM}} + F^i_{\mathrm{New}}]$$
$$+ \sum_k B^{\mathrm{New}}_k [\eta^k_{\mathrm{QCD}}]^{\mathrm{New}} V^k_{\mathrm{CKM}} [G^k_{\mathrm{New}}] \qquad (5)$$

with all the functions F^i_{SM}, F^i_{New} and G^k_{New} being real. Typical examples of new Dirac structures are the operators $(V-A) \otimes (V+A)$, $(S-P) \otimes (S \pm P)$ and $\sigma_{\mu\nu}(S-P) \otimes \sigma^{\mu\nu}(S-P)$ contributing to $B^0_{d,s} - \bar{B}^0_{d,s}$ mixings that become relevant in the MSSM with a large $\tan\beta$.

Class C

This class of models differs from class A through the presence of new complex phases in the Wilson coefficients of the usual SM operators. Contributions of new operators can be, however, neglected. An example is the MSSM with not too a large $\tan\beta$ and with non-diagonal elements in the squark mass matrices. This class can be summarized by

$$\mathrm{A(Decay)} = \sum_i B_i \eta^i_{\mathrm{QCD}} V^i_{\mathrm{CKM}} F_i(v) \qquad F_i(v) \ \ (\text{complex}). \qquad (6)$$

Class D

Here we group models with new complex phases, new operators and new flavour changing contributions which are not governed by the CKM matrix. As now the amplitudes are given by the most general expression (2), the phenomenology in this class of models is more involved than in the classes A–C [14,15]. Examples of models in class D are multi-Higgs models with complex phases in the Higgs sector, general SUSY models, models with spontaneous CP violation and left-right symmetric models.

Class E

Here we group models in which the unitarity of the three generation CKM matrix does not hold. Examples are four generation models and models with tree level FCNC transitions. If this type of physics is present, the unitarity triangle does not close. The most recent discussion of the possible violation of the unitarity of the three generation CKM matrix can be found in [16].

3 Class A: MFV Models

3.1 Model Independent Relations

One can derive a number of relations between various observables that do not depend on the functions $F_i(v)$ and consequently are universal within the class of the MFV models. Violation of any of these relations would be a signal of the presence of new operators and/or new weak complex phases. We list here the most interesting relations of this type.

1. A universal unitarity triangle (UUT) common to all MFV models can be constructed by using only observables that are independent of $F_i(v)$ [10]. In particular its two sides R_b and R_t [4] can be determined from $|V_{ub}/V_{cb}|$ and the ratio of $B_{d,s}^0 - \bar{B}_{d,s}^0$ mixing mass differences $\Delta M_{d,s}$, respectively:

$$R_b = 4.4 \left| \frac{V_{ub}}{V_{cb}} \right|, \quad R_t = 0.90 \left[\frac{\xi}{1.24} \right] \sqrt{\frac{18.4/ps}{\Delta M_s}} \sqrt{\frac{\Delta M_d}{0.50/ps}}, \tag{7}$$

where $\xi = 1.24 \pm 0.08$ [17] is a non-perturbative parameter, $\Delta M_d = (0.503 \pm 0.006)/ps$ and $\Delta M_s > 14.4/ps$. Moreover, the angle β of this triangle can be found from measurements of the time-dependent CP asymmetry $a_{\psi K_S}(t)$ with the result [18,19]

$$(\sin 2\beta)_{\psi K_S} = 0.736 \pm 0.049 \ . \tag{8}$$

Using (7) and (8) one finds the apex of the UUT placed within the larger ellipse in fig. 1 [20]. A similar analysis has been performed in [11].

2. Two theoretically clean relations are [21]

$$\frac{Br(B \to X_d \nu \bar{\nu})}{Br(B \to X_s \nu \bar{\nu})} = \left| \frac{V_{td}}{V_{ts}} \right|^2, \qquad \frac{Br(B_s \to \mu \bar{\mu})}{Br(B_d \to \mu \bar{\mu})} = \frac{\hat{B}_d}{\hat{B}_s} \frac{\tau(B_s)}{\tau(B_d)} \frac{\Delta M_s}{\Delta M_d}. \tag{9}$$

They do not involve the B_q–meson decay constants F_{B_q} and consequently contain substantially smaller hadronic uncertainties than the formulae for individual branching ratios [13]. The ratio $\hat{B}_s/\hat{B}_d$ is known from lattice calculations with a respectable precision [17]:

$$\frac{\hat{B}_s}{\hat{B}_d} = 1.00 \pm 0.03, \qquad \hat{B}_d = 1.34 \pm 0.12, \qquad \hat{B}_s = 1.34 \pm 0.12 \ . \tag{10}$$
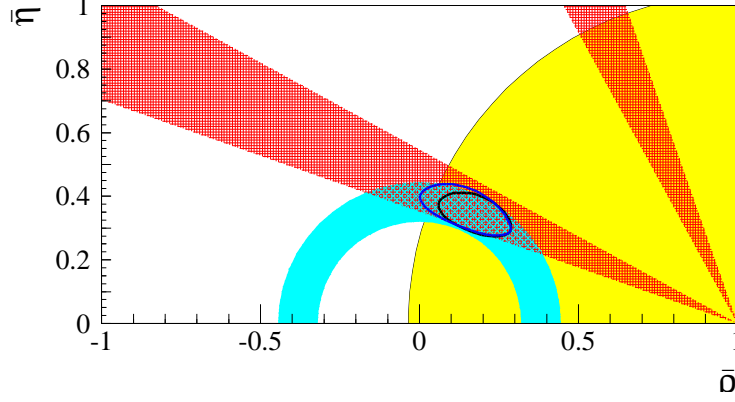
Fig. 1. The allowed 95% regions in the $(\bar{\varrho}, \bar{\eta})$ plane in the SM (narrower region) and in the MFV models (broader region) from the update of [20]. The individual 95% regions for the constraint from R_b, $\Delta M_d / \Delta M_s$ and $\sin 2\beta$ are also shown.

With a future precise measurement of ΔM_s, the second formula in (9) will give a precise prediction for the ratio of the branching ratios $Br(B_q \to \mu\bar{\mu})$.

3. It is possible to derive an accurate formula for $\sin 2\beta$ that depends only on the $K \to \pi\nu\bar{\nu}$ branching ratios and a calculable $\bar{P}_c(X) = 0.38 \pm 0.06$ [22,23,24]:

$$\sin 2(\beta - \beta_s) = \frac{2r_s}{1 + r_s^2}, \qquad r_s = \frac{\varepsilon_1\sqrt{B_1 - B_2} - \bar{P}_c(X)}{\varepsilon_2\sqrt{B_2}}, \tag{11}$$

where $\beta_s \approx -1°$ enters $V_{ts} = -|V_{ts}| \exp(-i\beta_s)$, $\varepsilon_i = \pm 1$ and

$$B_1 = \frac{Br(K^+ \to \pi^+\nu\bar{\nu})}{4.78 \cdot 10^{-11}}, \qquad B_2 = \frac{Br(K_{\mathrm{L}} \to \pi^0\nu\bar{\nu})}{2.09 \cdot 10^{-10}}. \tag{12}$$

In the MFV models $\varepsilon_1 = \varepsilon_2 = \mathrm{sgn}(X)$ [25], where X is the relevant master function.

4. With no weak phases beyond the CKM phase, we also expect

$$(\sin 2\beta)_{\pi\nu\bar{\nu}} = (\sin 2\beta)_{\psi K_S}, \qquad (\sin 2\beta)_{\phi K_S} \approx (\sin 2\beta)_{\psi K_S}, \tag{13}$$

with the accuracy of the last relation at the level of a few percent [26]. The confirmation of these two relations would be a very important test of the MFV idea. Indeed, in $K \to \pi\nu\bar{\nu}$ the phase β originates in the Z^0 penguin diagram, whereas in the case of $a_{\psi K_S}$ in the $B_d^0 - \bar{B}_d^0$ box diagrams. In the case of the asymmetry $a_{\phi K_S}$ it originates also in $B_d^0 - \bar{B}_d^0$ box diagrams, but the second relation in (13) could be spoiled by NP contributions in the decay amplitude for $B \to \phi K_S$ that is non-vanishing only at the one loop level.

Interestingly, the present data from Belle may indicate the violation of the second relation in (13), although the experimental situation is very unclear at present [27,28]:

$$(\sin 2\beta)_{\phi K_S} = \begin{cases} +0.45 \pm 0.43 \pm 0.07 \text{ (BaBar)} \\ -0.96 \pm 0.50^{+0.11}_{-0.09} \quad \text{(Belle)}, \end{cases} \tag{14}$$

A subset of theoretical papers addressing this issue is listed in [29].

5. An important consequence of (11)–(13) is the following one. For a given $\sin 2\beta$ extracted from $a_{\psi K_S}$ and $Br(K^+ \to \pi^+ \nu\bar{\nu})$ only two values of $Br(K_{\mathrm{L}} \to \pi^0 \nu\bar{\nu})$, corresponding to two signs of the master function $X(v)$, are possible in the full class of MFV models, independent of any new parameters present in these models [25]. Consequently, measuring $Br(K_{\mathrm{L}} \to \pi^0 \nu\bar{\nu})$ will either select one of these two possible values or rule out all MFV models.

6. As pointed out in [30] in most MFV models there exists a correlation between the zero $\hat{s}_0$ in the forward-backward asymmetry A_{FB} in $B \to X_s \mu^+ \mu^-$ and $Br(B \to X_s \gamma)$. We show this correlation in fig. 2.
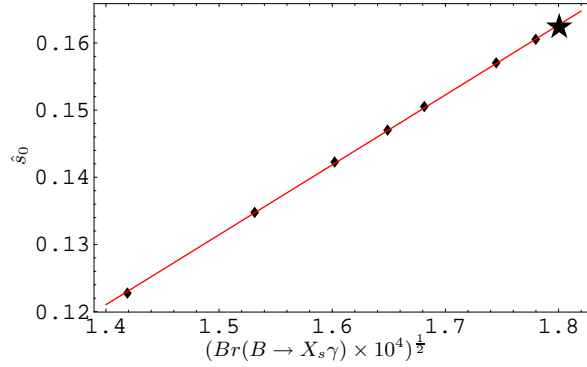

Fig. 2.   Correlation between $\sqrt{Br(B \to X_s\gamma)}$ and $\hat{s}_0$ [30]. The dots are the results in the ACD model (see below) with the compactification scale $200, 250, 300, 350, 400, 600$ and 1000 GeV and the star denotes the SM value.

7. Other correlations between various decays can be found in [25,31,32,33,34]. For instance there exists in addition to an obvious correlation between $K \to \pi\nu\bar{\nu}$ and $B \to X_q\nu\bar{\nu}$ also a correlation between ε'/ε and rare semileptonic B and K decays. A discussion of correlations between $B \to \pi K$ decays and rare decays within MFV models with enhanced Z^0 penguins can be found in [35].

3.2   Model Dependent Relations

$B_q \to \mu\bar{\mu}$ and ΔM_q The relations [21]

$$Br(B_q \to \mu\bar{\mu}) = 4.36 \cdot 10^{-10} \frac{\tau(B_q)}{\hat{B}_q} \frac{Y^2(v)}{S(v)} \Delta M_q, \qquad (q = s, d) \tag{15}$$

allow to predict $Br(B_{s,d} \to \mu\bar{\mu})$ in a given MFV model, characterized by $Y(v)$ and $S(v)$, with substantially smaller hadronic uncertainties than found by using directly the formulae for these branching ratios that suffer from large uncertainties due to F_{B_q}. In particular in the SM model we find [21]

$$Br(B_s \to \mu\bar{\mu}) = (3.4\pm0.5)\cdot10^{-9}, \quad Br(B_d \to \mu\bar{\mu}) = (1.00\pm0.14)\cdot10^{-10}, \quad (16)$$

where $\overline{m}_t(m_t) = (167 \pm 5)\,\text{GeV}$, the lifetimes from [17], $\hat{B}_q$ in (10), $\Delta M_d = (0.503 \pm 0.006)/\text{ps}$ and as an example $\Delta M_s = (18.0 \pm 0.5)/\text{ps}$, have been used. These results are substantially more accurate than the ones found in the literature in the past but by orders of magnitude below the experimental upper bounds from CDF(D0) and Belle [36,37].

$Br(K^+ \to \pi^+\nu\bar{\nu})$, $\Delta M_d/\Delta M_s$ and β. In [38] an upper bound on $Br(K^+ \to \pi^+\nu\bar{\nu})$ in terms of $\Delta M_d/\Delta M_s$ has been derived within the SM. It has been subsequently cast into a useful relation between $Br(K^+ \to \pi^+\nu\bar{\nu})$, $\Delta M_d/\Delta M_s$ and β in [39]. In any MFV model this relation reads

$$Br(K^+ \to \pi^+\nu\bar{\nu}) = 7.54 \cdot 10^{-6}|V_{cb}|^4 X_{\text{eff}}^2(v) \qquad (17)$$

$$X_{\text{eff}}^2(v) = X^2(v)\left[\sigma R_t^2 \sin^2\beta + \frac{1}{\sigma}\left(R_t\cos\beta + \frac{\lambda^4 P_c(X)}{|V_{cb}|^2 X(v)}\right)^2\right], \qquad (18)$$

where $\sigma = 1/(1 - \lambda^2/2)^2$, $\lambda = 0.224$, $P_c(X) = 0.39 \pm 0.06$ and R_t is given in (7). This formula is theoretically rather clean and does not involve hadronic uncertainties except for ξ in (7) and to a lesser extent in $|V_{cb}|$.

3.3   Maximal Enhancements

How large could various branching ratios in the MFV models be? A detailed numerical analysis of this question is beyond the scope of this presentation but, assuming that the dominant NP effects in rare K and B decays come from enhanced Z^0 penguins hidden in the master functions $X(v)$, $Y(v)$ and $Z(v)$, bounding this enhancement by the Belle and BaBar data on $B \to X_s l^+l^-$ [40] and setting all other parameters at their central values, we find the results in column MFV of table 1 where also the SM results are shown. While somewhat higher values of branching ratios can still be obtained when the input parameters are varied, this exercise shows that enhancements of branching ratios in the MFV models by more than factors of six relative to the SM should not be expected. A similar analysis in a different spirit and with a different set of input parameters can be found in [11].

Table 1. Example of branching ratios for rare decays in the MFV and the SM.

Branching Ratios	MFV	SM
$Br(K^+ \rightarrow \pi^+ \nu\bar{\nu}) \times 10^{11}$	19.1	8.0
$Br(K_{\rm L} \rightarrow \pi^0 \nu\bar{\nu}) \times 10^{11}$	9.9	3.2
$Br(K_{\rm L} \rightarrow \mu^+\mu^-)_{\rm SD} \times 10^9$	3.5	0.9
$Br(K_{\rm L} \rightarrow \pi^0 e^+e^-)_{\rm CPV} \times 10^{11}$	4.9	3.2
$Br(B \rightarrow X_s \nu\bar{\nu}) \times 10^5$	11.1	3.6
$Br(B \rightarrow X_d \nu\bar{\nu}) \times 10^6$	4.9	1.6
$Br(B_s \rightarrow \mu^+\mu^-) \times 10^9$	19.4	3.9
$Br(B_d \rightarrow \mu^+\mu^-) \times 10^{10}$	6.1	1.2

3.4   MFV and Universal Extra Dimensions

A detailed analysis of all rare and radiative K and B decays and of $\Delta M_{d,s}$ in a particular model with one universal extra dimension (ACD) [41] has been presented in [30]. The nice feature of this extension of the SM is the presence of only one additional parameter, the compactification scale. This feature allows a unique pattern of various enhancements and suppressions relative to the SM expectations. Our analysis shows that all the present data on FCNC processes are consistent with the compactification scale as low as 300 GeV, implying that the Kaluza–Klein particles could in principle be found already at the Tevatron. Possibly, the most interesting results of our analysis is the sizable downward shift of the zero ($\hat{s}_0$) in the $A_{\rm FB}$ asymmetry in $B \rightarrow X_s \mu^+\mu^-$ and the suppression of $Br(B \rightarrow X_s\gamma)$ (also found in [42]) that are correlated as shown in fig. 2. Note that a measurement of $\hat{s}_0$ that is higher than the SM estimate would automatically exclude this model as there is no compactification scale for which this could be satisfied. The impact of the Kaluza-Klein particles on $\Delta M_{s,d}$ and on the electroweak precision observables can also be found in [43] and [41,44], respectively.

4   Class B: MSSM at Large $\tan\beta$

An example of a model in this class is the MSSM with large $\tan\beta$ but without any relevant contributions from new flavour violating interactions coming from the non-diagonal elements in squark mass matrices when these are given in the quark mass eigenstate basis. In these models the dominant new effects in K and B decays come from very strongly enhanced flavour changing neutral (FCNC) Higgs couplings to the down quarks. These couplings are generated only at one loop level but being proportional to $(\tan\beta)^2$ become very important for $\tan\beta \geq 30$.

The presence of the enhanced FCNC Higgs couplings implies in turn important contributions of new operators in the effective theory that are strongly suppressed in models of class A. In particular the operators

$$O_S = m_b(\overline{b_R}s_L)(\bar{\mu}\mu), \qquad O_P = m_b(\overline{b_R}s_L)(\bar{\mu}\gamma_5\mu) \tag{19}$$

fully dominate the branching ratios for $B_{s,d} \to \mu^+\mu^-$ [45] when $\tan\beta \geq 30$. An approximate formula for $Br(B_s \to \mu^+\mu^-)$ is given then by [46,47]

$$Br(B_s \to \mu^+\mu^-) \approx 3.5 \cdot 10^{-5} \left[\frac{\tan\beta}{50}\right]^6 \left[\frac{m_t}{M_A}\right]^4 F(\varepsilon_i, \tan\beta) , \tag{20}$$

$$F(\varepsilon_i, \tan\beta) = \left[\frac{16\pi^2\varepsilon_Y}{(1 + \tilde{\varepsilon}_3 \tan\beta)(1 + \tilde{\varepsilon}_0 \tan\beta)}\right]^2 , \tag{21}$$

where ε_i, depending on the SUSY parameters, are at most $\mathcal{O}(10^{-2})$ and we have set $\tau(B_s)$, F_{B_s} and $|V_{ts}|$ at their central values. M_A is the mass of the pseudoscalar A. The expression for $Br(B_d \to \mu^+\mu^-)$ is obtained by using

$$\frac{Br(B_d \to \mu^+\mu^-)}{Br(B_s \to \mu^+\mu^-)} = \left[\frac{\tau(B_d)}{\tau(B_s)}\right] \left[\frac{F_{B_d}}{F_{B_d}}\right]^2 \left|\frac{V_{td}}{V_{ts}}\right|^2 \left[\frac{M_{B_d}}{M_{B_d}}\right]^5 , \tag{22}$$

which differs slightly from the usual MFV formula in that the last factor has the power five instead of two. On the other hand the branching ratios themselves can still be enhanced by almost a factor of 500. If this is indeed the case, $B_{s,d} \to \mu^+\mu^-$ should be observed already at Tevatron and B factories, respectively.

The enhanced neutral Higgs couplings and more generally large $\tan\beta$ effects can play also a significant role in ΔM_s [47] inducing in particular four–fermion operators

$$Q_1^{LR} = (\overline{b_L}\gamma_\mu s_L)(\overline{b_R}\gamma^\mu s_R), \qquad Q_2^{LR} = (\overline{b_R}s_L)(\overline{b_L}s_R), \tag{23}$$

whose Wilson coefficients are negligible in the SM. One finds then

$$\Delta M_s = (\Delta M_s)_{\text{SM}}(1 + f_s) \approx (\Delta M_s)_{\text{SM}} - |\Delta M_s|^{\text{DP}}, \tag{24}$$

where the new contributions come dominantly from double Higgs penguins (DP), as indicated above. Being proportional to $m_q m_b \tan^4\beta$ for ΔM_q, these contributions can be neglected in ΔM_d that for large $\tan\beta$ is very close to the SM estimate. It turns out that ΔM_s is suppressed for any choice of supersymmetric parameters ($f_s < 0$) with the size of suppression dependent strongly on the stop mixing, M_A and $\tan\beta$.

As a consequence of the mismatch between Higgs contributions to ΔM_d and ΔM_s, the MFV formula for R_t in (7) is modified to

$$R_t = 0.90 \left[\frac{\xi}{1.24}\right] \sqrt{\frac{18.4/ps}{\Delta M_s}} \sqrt{\frac{\Delta M_d}{0.50/ps}} \sqrt{1 + f_s}. \tag{25}$$

However, most interesting is the correlation of the enhanced neutral Higgs effects in $B_{s,d} \to \mu^+\mu^-$ and ΔM_s, that is independent of $F(\varepsilon_i, \tan\beta)$ in (21):

$$Br(B_s \to \mu^+\mu^-) \approx 10^{-6} \left[\frac{\tan\beta}{50}\right]^2 \left[\frac{200\,\text{GeV}}{M_A}\right]^2 \left[\frac{|\Delta M_s|^{\text{DP}}}{2.1/\text{ps}}\right]. \qquad (26)$$

Consequently a strong enhancement of $B_{s,d} \to \mu^+\mu^-$ implies a significant suppression of ΔM_s. This means that, in this scenario, an observation of $B_{s,d} \to \mu^+\mu^-$ at the level of $\mathcal{O}(10^{-7})$ and $\mathcal{O}(10^{-8})$ respectively should be accompanied by ΔM_s below the SM estimates. On the other hand, if $(\Delta M_s)_{\text{exp}} > (\Delta M_s)_{\text{SM}}$ is found, this scenario will be excluded and the observation of $B_{s,d} \to \mu^+\mu^-$ at this level would point toward other flavour violating sources, coming for instance from non-diagonal elements in the squark mass matrices [48,49].

The difficult task in testing this scenario will be to demonstrate whether the measured value $(\Delta M_s)_{\text{exp}}$ is indeed smaller or larger than $(\Delta M_s)_{\text{SM}}$. To this end a significant reduction of the uncertainties in the non-perturbative parameters is required. On the other hand an enhancement of $B_{s,d} \to \mu^+\mu^-$ by one or two orders of magnitude with respect to the SM estimates in (16) would be truly spectacular independently of the situation concerning ΔM_s.

Other interesting correlations are the ones between the ratios $Br(B \to H\mu^+\mu^-)/Br(B \to He^+e^-)$ $(H = K^{(*)}, X_s)$ and $Br(B_s \to \mu^+\mu^-)$ [50].

5   Class C: New Weak Phases

5.1   Preliminaries

In this class of models the dominant operators are as in class A but the master functions become now complex quantities. If the new weak phases are large, the deviations from the SM can be spectacular, as we will see below.

5.2   Weak Phases in $\Delta F = 2$ Transitions

In the MFV scenario of Section 3 the NP effects enter universally in $K^0 - \bar{K}^0$, $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ mixing through the single real function $S(v)$, implying strong correlations between new physics effect in the $\Delta F = 2$ observables of K and B decays. When new complex weak phases are present, the situation could be more involved, with $S(v)$ replaced by

$$S_K(v) = |S_K(v)|e^{i\theta_K}, \quad S_d(v) = |S_K(v)|e^{i\theta_d}, \quad S_s(v) = |S_K(v)|e^{i\theta_s}, \quad (27)$$

for $K^0 - \bar{K}^0$, $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ mixing, respectively. If these three functions are different from each other, some universal properties found in class A are lost. In addition the mixing induced CP asymmetries in B decays will not measure the angles of the UT but only sums of these angles and of θ_i. Yet, within each class of K, B_d and B_s decays, the NP effects of this sort will be universal. Scenarios of this type have been considered for instance in [51].

5.3 Weak Phases in $\Delta F = 1$ Transitions

New weak phases could enter also decay amplitudes. As now these effects enter in principle differently in each decay, the situation can be very involved with many free parameters, no universal effects and little predictive power.

Here I will only discuss one scenario, discussed first in [31,32,52,53] and recently in the context of a simultaneous analysis of prominent non–leptonic B decays like $B \to \pi\pi$, $B \to \pi K$, $B \to \psi K_S$ and $B \to \phi K_S$ and equally prominent rare decays like $K \to \pi\nu\bar{\nu}$, $K_L \to \pi^0 e^+ e^-$, $B_{s,d} \to \mu^+\mu^-$, $B \to X_{s,d} e^+ e^-$ and ε'/ε in [23,24]. It is the scenario of enhanced Z^0 penguins with a large complex weak phase in which the only modification with respect to class A is the replacement in the Z^0 penguin function $C(v) \to |C(v)|e^{i\theta_C}$ that makes the master functions $X(v)$, $Y(v)$ and $Z(v)$ complex quantities:

$$X(v) = |X(v)|e^{i\theta_X}, \qquad Y(v) = |Y(v)|e^{i\theta_Y}, \qquad Z(v) = |Z(v)|e^{i\theta_Z}. \quad (28)$$

The magnitudes and phases of these three functions are correlated with each other as they depend only on $|C(v)|e^{i\theta_C}$ and other smaller contributions, that can be set to their SM values.

This new analysis has been motivated by the experimental situation in $B \to \pi\pi$ and $B \to \pi K$ decays described below. While our analysis does not rely on a particular model with the properties specified above, concrete models with enhanced CP-violating Z^0-mediated FCNC couplings generated either at the one-loop level or even at the tree level have been discussed in the literature. They are reviewed in [31,32,52,53], in particular in the last of these papers; see also [54]. Also models with Z'-mediated FCNCs could be put in this class, provided their contributions can effectively be absorbed in the function $C(v)$. For a recent analysis, see [55].

5.4 The $B \to \pi\pi$ Puzzle

The BaBar and Belle collaborations have very recently reported the observation of $B_d \to \pi^0\pi^0$ decays with CP-averaged branching ratios of $(2.1 \pm 0.6 \pm 0.3) \times 10^{-6}$ and $(1.7 \pm 0.6 \pm 0.2) \times 10^{-6}$, respectively [56,57]. These measurements represent quite some challenge for theory. For example, in a recent state-of-the-art calculation [58] within QCD factorization (QCDF) [59], a branching ratio that is about six times smaller is favoured, whereas the calculation of $B_d \to \pi^+\pi^-$ points towards a branching ratio about two times larger than the current experimental average. On the other hand, the calculation of $B^+ \to \pi^+\pi^0$ reproduces the data rather well. This "$B \to \pi\pi$ puzzle" is reflected by the following quantities:

$$R_{+-}^{\pi\pi} \equiv 2 \left[\frac{Br(B^\pm \to \pi^\pm\pi^0)}{Br(B_d \to \pi^+\pi^-)} \right] \frac{\tau_{B_d^0}}{\tau_{B^+}} = 2.12 \pm 0.37 \quad (29)$$

$$R_{00}^{\pi\pi} \equiv 2 \left[\frac{Br(B_d \to \pi^0\pi^0)}{Br(B_d \to \pi^+\pi^-)} \right] = 0.83 \pm 0.23. \quad (30)$$

The central values calculated in QCDF [58] are $R_{+-}^{\pi\pi} = 1.24$ and $R_{00}^{\pi\pi} = 0.07$.

5.5   The $B \to \pi K$ Puzzle

In the $B \to \pi K$ system, the CLEO, BaBar and Belle collaborations have measured the following ratios of CP-averaged branching ratios [60]:

$$R_{\rm c} \equiv 2 \left[\frac{Br(B^{\pm} \to \pi^0 K^{\pm})}{Br(B^{\pm} \to \pi^{\pm} K^0)} \right] = 1.17 \pm 0.12 \tag{31}$$

$$R_{\rm n} \equiv \frac{1}{2} \left[\frac{Br(B_d \to \pi^{\mp} K^{\pm})}{Br(B_d \to \pi^0 K)} \right] = 0.76 \pm 0.10, \tag{32}$$

with numerical values following from [61]. As noted in [62], the pattern of $R_{\rm c} > 1$ and $R_{\rm n} < 1$, which is now consistently favoured by the separtate BaBar, Belle and CLEO data, is actually puzzling in the framework of QCD factorization [58] that gives typically $R_{\rm c} \approx R_{\rm n} \approx 1.15$. This is clearly seen in the $(R_{\rm n}, R_{\rm c})$ plot in fig. 3 [23], to which we will return below. On the other hand,

$$R \equiv \left[\frac{Br(B_d \to \pi^{\mp} K^{\pm})}{Br(B^{\pm} \to \pi^{\pm} K)} \right] \frac{\tau_{B^+}}{\tau_{B_d^0}} = 0.91 \pm 0.07 \tag{33}$$

does not show any anomalous behaviour. Since $R_{\rm c}$ and $R_{\rm n}$ are affected significantly by colour-allowed EW penguins, whereas this is not the case of R, this "$B \to \pi K$ puzzle" may be a manifestation of NP in the EW penguin sector [23,24,35,62], offering an attractive avenue for physics beyond the SM to enter the $B \to \pi K$ system [63,64].

5.6   The Analysis of [23,24]

In view of significant experimental uncertainties, none of these exciting results is conclusive at the moment, but it is legitimate and interesting to take them seriously and to search for possible origins of these "signals" for deviations from the SM expectations. As we are dealing here with non-leptonic decays, the natural question arises whether these signals originate in the NP contributions or/and result from our insufficient understanding of hadron dynamics. The purpose of [23,24] was to develop a strategy which would allow us to address the $B \to \pi\pi$ and $B \to \pi K$ puzzles in a systematic manner once the experimental data on these decays improve. In order to illustrate this strategy in explicit terms, we considered a simple extension of the SM in which NP enters dominantly through enhanced CP-violating Z^0 penguins. As we will see below, this choice is dictated by the pattern of the data on the $B \to \pi K$ observables and the great predictivity of this scenario. It was first considered in [31,32,52] to study correlations between rare K decays and the ratio ε'/ε, and was generalized to rare B decays in [53]. Extending these considerations to non-leptonic B-meson decays, allowed us to confront this NP scenario with many more experimental results. Our strategy consists of three interrelated steps, and has the following logical structure:
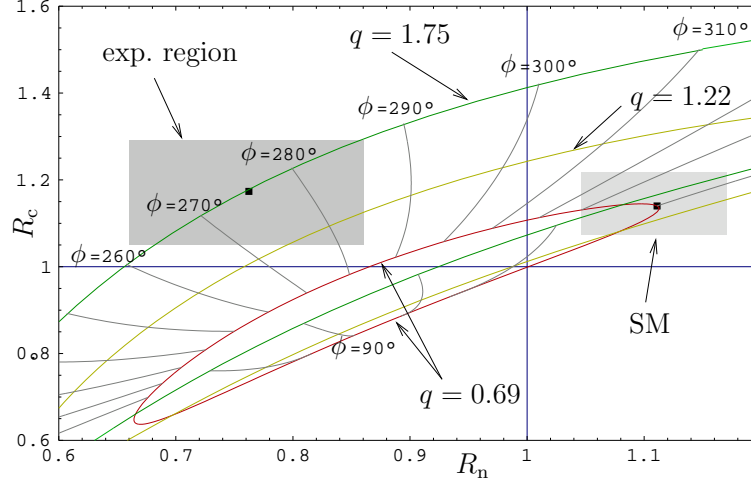
Fig. 3. The situation in the R_{n}–R_{c} plane [23]. We show contours for values of $q = 0.69$, $q = 1.22$ and $q = 1.75$ and $\phi \in [0°, 360°]$. The ranges from (31) and (32) (experiment) as well as from the SM are indicated in grey.

Step 1:
Since $B \to \pi\pi$ decays and the usual analysis of the UT are only insignificantly affected by EW penguins, the $B \to \pi\pi$ system can be described as in the SM. Employing the $SU(2)$ isospin flavour symmetry of strong interactions and the information on γ from the UT fits [17], we could extract the relevant hadronic $B \to \pi\pi$ parameters, and find large non-factorizable contributions, in variance with the QCD factorization approach. Having these parameters at hand, we could then also predict the direct and mixing-induced CP asymmetries of the $B_d \to \pi^0\pi^0$ channel. A future measurement of one of these observables allows a determination of γ.

Step 2:
Using the $SU(3)$ flavour symmetry and plausible dynamical assumptions, we could subsequently determine the hadronic $B \to \pi K$ parameters through the $B \to \pi\pi$ analysis, and calculate the $B \to \pi K$ observables in the SM. Interestingly, we found agreement with the pattern of the B-factory data for those observables where EW penguins play only a minor rôle. On the other hand, the observables receiving significant EW penguin contributions did *not* agree with the experimental picture, thereby suggesting NP in the EW penguin sector. Parametrizing the EW contributions to $B \to \pi K$ by $qe^{i\phi}$ that is related to the Z^0-penguin function $C(v)$ through [35]

$$C(v) = |C(v)|e^{i\theta_C} = 2.35\,\bar{q}e^{i\phi} - 0.82, \quad \bar{q} = q\left[\frac{|V_{ub}/V_{cb}|}{0.086}\right], \tag{34}$$

we have demonstrated [23,24] that one can describe all the currently available $B \to \pi K$ data provided

$$q = 1.75^{+1.27}_{-0.99}, \quad \phi = -(85^{+11}_{-14})^{\circ}, \tag{35}$$

to be compared with $q = 0.69$ [65] and $\phi = 0$ in the SM. In particular as seen in fig. 3 the values in (35) allow to describe properly the data in (31) and (32). The requirement of a large CP-violating NP phase around -90° is the most interesting result of this study. A crucial future test of this scenario will be provided by the CP-violating $B_d \to \pi^0 K_{\mathrm{S}}$ observables, which we could predict. Moreover, we could obtain valuable insights into $SU(3)$-breaking effects, which support our working assumptions, and could also determine the UT angle γ, that is in agreement with the UT fits.

Step 3:
In turn, the sizeably enhanced EW penguins with their large CP-violating NP phase have important implications for rare K and B decays as well as ε'/ε. Interestingly, several predictions differ significantly from the SM expectations and should easily be identified once the data improve.

Including the constraint from $B \to X_s l^+ l^-$ [40], that selects $q \leq 1.0$ in (35), the most interesting results of this study, presented in [23,24] are:
a) For the very clean $K \to \pi \nu \bar{\nu}$ decays, we find

$$\begin{aligned} Br(K^+ \to \pi^+ \nu \bar{\nu}) &= (7.5 \pm 2.1) \cdot 10^{-11}, \\ Br(K_{\mathrm{L}} \to \pi^0 \nu \bar{\nu}) &= (3.1 \pm 1.0) \cdot 10^{-10}, \end{aligned} \tag{36}$$

to be compared with the SM predictions, for which we find $(8.0 \pm 1.1) \times 10^{-11}$ and $(3.2 \pm 0.5) \times 10^{-11}$ in the ballpark of other estimates [66], respectively. On the experimental side the results of the AGS E787 [67] collaboration and KTeV [68] are $Br(K^+ \to \pi^+ \nu \bar{\nu}) = (15.7^{+17.5}_{-8.2}) \times 10^{-11}$ and $Br(K_{\mathrm{L}} \to \pi^0 \nu \bar{\nu}) < 5.9 \times 10^{-7}$, respectively. The enhancement of $Br(K_{\mathrm{L}} \to \pi^0 \nu \bar{\nu})$ by one order of magnitude and the pattern in (36) are dominantly the consequences of $\beta_X = \beta - \theta_X \approx 111^{\circ}$ with θ_X defined in (28). Indeed,

$$\frac{Br(K_{\mathrm{L}} \to \pi^0 \nu \bar{\nu})}{Br(K_{\mathrm{L}} \to \pi^0 \nu \bar{\nu})_{\mathrm{SM}}} = \left| \frac{X}{X_{\mathrm{SM}}} \right|^2 \left[\frac{\sin \beta_X}{\sin \beta} \right]^2 \tag{37}$$

$$\frac{Br(K_{\mathrm{L}} \to \pi^0 \nu \bar{\nu})}{Br(K^+ \to \pi^+ \nu \bar{\nu})} \approx 4.4 \times (\sin \beta_X)^2 \approx (4.2 \pm 0.2). \tag{38}$$

Interestingly, the above ratio turns out to be very close to its absolute upper bound in [69]. Fig. 4 illustrates nicely these features. A spectacular implication of these findings is a strong violation of $(\sin 2\beta)_{\pi \nu \bar{\nu}} = (\sin 2\beta)_{\psi K_{\mathrm{S}}}$ [22], which is valid in the SM and any model with minimal flavour violation as discussed in Section 3. Indeed, we find

$$(\sin 2\beta)_{\pi \nu \bar{\nu}} = \sin 2\beta_X = -(0.69^{+0.23}_{-0.41}), \tag{39}$$

in striking disagreement with $(\sin 2\beta)_{\psi K_{\mathrm{S}}} = 0.74 \pm 0.05$ in (8).

b) Another implication is the large branching ratio

$$Br(K_{\mathrm{L}} \to \pi^0 e^+ e^-) = (7.8 \pm 1.6) \times 10^{-11}, \tag{40}$$

which is governed by direct CP violation in this scenario. On the other hand, the SM result $(3.2^{+1.2}_{-0.8}) \times 10^{-11}$ [70] is dominated by the indirect CP violation. Next, the integrated forward–backward CP asymmetry for $B_d \to K^* \mu^+ \mu^-$ [53] can be very large in view of $\theta_Y \approx -100°$ as it is given by

$$A_{\mathrm{FB}}^{\mathrm{CP}} = (0.03 \pm 0.01) \times \tan \theta_Y. \tag{41}$$

c) Next, $Br(B \to X_{s,d} \nu \bar{\nu})$ and $Br(B_{s,d} \to \mu^+ \mu^-)$ are enhanced by factors of 2 and 5, respectively. The impact on $K_{\mathrm{L}} \to \mu^+ \mu^-$ is rather moderate.

d) As emphasized in [31], enhanced Z^0 penguins may play an important rôle in ε'/ε. The enhanced value of C and its large negative phase suggested by the $B \to \pi K$ analysis require a significant enhancement of the relevant hadronic matrix element of the QCD penguin operator Q_6, with respect to the one of the EW penguin operator Q_8, to be consistent with the ε'/ε data.

e) We have also explored the implications for the decay $B_d \to \phi K_{\mathrm{S}}$ [23]. Large hadronic uncertainties preclude a precise prediction, but assuming that the sign of the cosine of a strong phase agrees with factorization, we find that $(\sin 2\beta)_{\phi K_{\mathrm{S}}} > (\sin 2\beta)_{\psi K_{\mathrm{S}}}$. This pattern is qualitatively different from the present B-factory data [27], which are, however, not yet conclusive as seen in (14). On the other hand, a future confirmation of this pattern could be another signal of enhanced CP-violating Z^0 penguins at work.

6   Shopping List

We have seen that each of the NP scenarios discussed above had some specific features not shared by other scenarios and with a sufficient number of measurements it should be possible to distinguish between them, eventually selecting one of them or demonstrating the necessity for going to scenarios in classes D and E.

There is a number of questions which I hope will be answered in the coming years:

- Probably the most important at present is the clarification of the discrepancy between Belle [28] and BaBar [27] in the measurement of $(\sin 2\beta)_{\phi K_S}$. The confirmation of the significant departure of $(\sin 2\beta)_{\phi K_S}$ from the already accurate value of $(\sin 2\beta)_{\psi K_S}$, would be a clear signal of new physics that cannot be accomodated within classes A and B. Also in a particular scenario of class C, just discussed, one finds $(\sin 2\beta)_{\phi K_S} > (\sin 2\beta)_{\psi K_S}$ and consequently $(\sin 2\beta)_{\phi K_{\mathrm{S}}}$ is of the same magnitude as the central value found by the Belle collaboration but of *opposite* sign! Thus it is likely that the confirmation of the Belle result will require the models of class D.
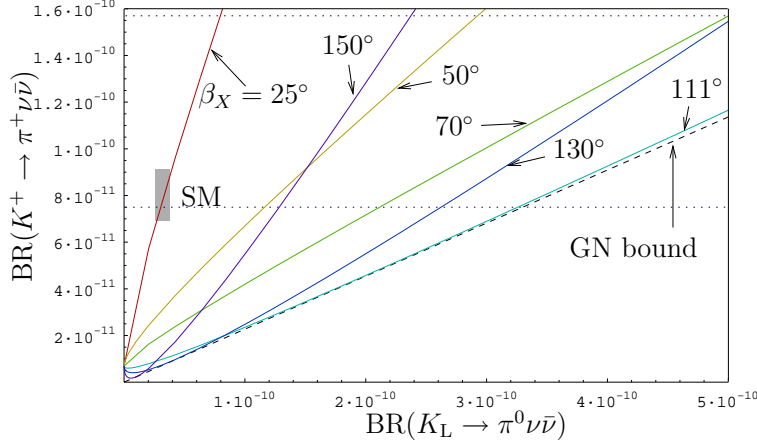
Fig. 4. $Br(K^+ \to \pi^+\nu\bar{\nu})$ as a function of $Br(K_L \to \pi^0\nu\bar{\nu})$ for various values of β_X [23]. The dotted horizontal lines indicate the lower part of the experimental range and the grey area the SM prediction. We also show the bound of [69].

- Also very important are the measurements of $Br(B_{d,s} \to \mu^+\mu^-)$ and ΔM_s. The possible enhancements of $Br(B_{d,s} \to \mu^+\mu^-)$ by factors as high as 500, discussed in Section 4, are the largest enhancements in the field of K and B decays, that are still consistent with all available data. The measurement of ΔM_s is, on the other hand, very important as it may have a considerable impact on the determination of the unitarity triangle. Finding ΔM_s below $(\Delta M_s)_{SM}$ and $Br(B_{d,s} \to \mu^+\mu^-)$ well above the SM expectations would be a nice confirmation of a SUSY scenario with a large $\tan\beta$ that we discussed in Section 4.
- The improved measurements of several $B \to \pi\pi$ and $B \to \pi K$ observables are very important in order to see whether the theoretical approaches like QCDF [59], PQCD [71] and SCET [72] in addition to their nice theoretical structures are also phenomenologically useful. On the other hand, independently of the outcome of these measurements, the pure phenomenological strategy [23,24] presented in Section 5, will be useful in correlating the experimental results for $B \to \pi\pi$ and $B \to \pi K$ with those for rare K and B decays, $B_d \to \phi K_S$ and ε'/ε.
- Assuming that the future more accurate data on $B \to \pi\pi$ and $B \to \pi K$ will not modify significantly the presently observed pattern in these decays, the scenario of enhanced Z^0 penguins with a new large complex weak phase will remain to be an attractive possibility. While the enhancement of $Br(K_L \to \pi^0\nu\bar{\nu})$ by one order of magnitude would be very welcome to our experimental colleagues and $(\sin 2\beta)_{\pi\nu\bar{\nu}} < 0$ would be a very spectacular signal of NP, even more moderate departures of this sort

from the SM and the MFV expectations could be easily identified in the very clean $K \to \pi \nu \bar{\nu}$ decays as clear signals of NP.

- The improved measurements of $Br(B \to X_s l^+ l^-)$ and $Br(K^+ \to \pi^+ \nu \bar{\nu})$ in the coming years will efficiently bound the possible enhancements of Z^0 penguins, at least within the scenarios A–C discussed here.

- Also very important is an improved measurement of $Br(B \to X_s \gamma)$ as well as the removal of its sensitivity to μ_c in $m_c(\mu_c)$ through a NNLO calculation. This would increase the precision on the MFV correlation between $Br(B \to X_s \gamma)$ and the zero $\hat{s}_0$ in the forward–backward asymmetry $A_{\mathrm{FB}}(\hat{s})$ in $B \to X_s l^+ l^-$. A 20% suppression of $Br(B \to X_s \gamma)$ with respect to the SM accompanied by a downward shift of $\hat{s}_0$ would be an interesting confirmation of the correlation in question and consistent with the effects of universal extra dimensions with a low compactification scale of order few hundred GeV. On the other hand finding no zero in $A_{\mathrm{FB}}(\hat{s})$ would likely point towards flavour violation beyond the MFV.

- Finally, improved bounds and/or measurements of processes not existing or very strongly suppressed in the SM, like various electric dipol moments and FCNC transitions in the charm sector will be very important in the search for new physics. The same applies to $\mu \to e\gamma$ and generally lepton flavour violation.

We could continue this list for ever, in particular in view of the expected progress at Belle and Babar, charm physics at Cornell, experimental program at LHCb, BeTeV and the planned rare K physics experiments. But the upper bound on the maximal number of pages for my contribution has been already significantly violated which is a clear signal that I should conclude here. The conclusion is not unexpected: in this decade, it will be very exciting to follow the development in this field and to monitor the values of various observables provided by our experimental colleagues by using the strategies presented here and other strategies found in the rich literature.

Acknowledgements

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